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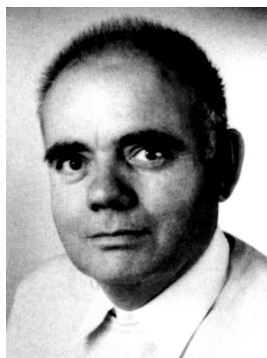
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Authors Bio



Arthur Engel (1928 – 2022) was a German mathematics teacher, educationalist, and prolific author. He was very active in the German National Mathematics Competition, and quite a few of the problems in this book were inspired by those competitions.

In 1990 Engel was awarded the Federal Cross of Merit by the Minister of Education of West Germany. In 1991 he was one of the first three recipients of the David Hilbert Award of the World Federation of National Mathematics Competitions.

With his 1984 'Elementary Mathematics from an Algorithmic Standpoint', Engel became one of the first educators to propose that mathematics in schools should focus on the concept of algorithm. This was followed in 1993 by the original version of this text, 'Exploring Mathematics with Your Computer', which excitingly illustrated the value of programming for teaching mathematics, and math as a source of informative programming problems.

Andrew Davison (1962 -) is a British computer science teacher and author, with a long career in academia. For many happy years, he has taught numerous topics in the Department of Computer Engineering, Faculty of Engineering, Prince of Songkla University (PSU) in Hat Yai, Thailand (<http://coe.psu.ac.th/~ad/>).

I first encountered Engel's work in the late 1980s through his wonderful 'Elementary Mathematics from an Algorithmic Standpoint', and later 'Exploring Mathematics with Your Computer', which offered many fascinating problems suitable for students of discrete math and algorithms. The math content of these books is timeless, but the programming examples, rooted in BASIC and Pascal, became a major hindrance to their use. I offered to revise 'Exploring Mathematics with Your Computer' making it relevant for modern programming students.