Appendix C

Summary of Notation and Formulas

- $a|b$: *a divides b*, i.e. b is a *multiple* of a for integers a, b.
- $a..b$: the set $\{a, a+1, ..., b-1, b\}$ of integers.
- $\lfloor x \rfloor$: *floor* of x, i.e. the largest integer $\leq x$. Usually denoted by $[x]$.
- $\lceil x \rceil$: *ceiling* of x , i.e. the smallest integer $\geq x$.
- gcd(a, b): *greatest common divisor* of the integers a, b.
- lcm(a, b): *least common multiple* of the integers a, b.
- $p \Rightarrow q$: The statement p *implies* the statement q.
- $p \Leftrightarrow q$: p is true *if and only if* q is true.
- $\mapsto$: mapping or function symbol. $a \mapsto b$: a is mapped into b.
- $f(n) \sim g(n)$: $f(n)$ is *asymptotically equal* to $g(n)$, i.e. $f(n)/g(n) \rightarrow 1$ for $n \rightarrow \infty$.
- $f(n) = O(g(n))$: There is a constant C such that $|f(n)| \leq Cg(n)$.
- *Harmonic series*: $1 + \frac{1}{2} + \frac{1}{3} + \cdots$. The n-th *harmonic number* $H_n = 1 + \frac{1}{2} + \cdots \frac{1}{n}$ is the n-th partial sum of the series.

- $\phi(n) = n \prod_p (1 - 1/p)$, where the product ranges over all distinct prime factors of n. This is the number of integers in $1..n$ which are relatively prime to n. $\phi(n)$ is called *Euler's phi function*.
- $\sum_{n \geq 1} 1/n^2 = \pi^2/6$, a classic result due to Euler.
- $\binom{n}{s}$: the number of s-subsets of an n-set

$$\binom{n}{s} = \frac{n}{s}\binom{n-1}{s-1} = \frac{n(n-1)...(n-s+1)}{s(s-1)...1} = \frac{n!}{s!(n-s)!}$$

  is called *n choose s*, or the *binomial coefficient n over s*. Here $n!$, pronounced *n factorial*, is the product $1 \cdot 2 \cdot 3 \cdots n$, with $0! = 1$.